

be pointed out that the former is inherently exact since it satisfies directly the boundary conditions of unconfined flow. The method was demonstrated using an incompressible wind tunnel flow and therefore is not limited to small perturbations in this Mach number regime. At high subsonic Mach numbers the method still applies as long as the inner region contains at worst weak shocks. Performing flowfield calculations in inner and outer regions at these Mach numbers would be significantly more complicated and require much more computer time. It is not clear to the writer just how drastically it would change the description of the model. If the flow in the inner region were nearly linear the iterations could still be performed using increments to boundary conditions. The extension of the writer's methods to these higher Mach numbers is being studied. The extension of the computer programs to make more detailed wall correction calculations at the model also has not yet been done. At this point, then, it appears that the two methods would be comparable at high subsonic Mach numbers but the Sears method would be more accurate for low speed wind tunnels.

## Comment on "Improved Solutions to the Falkner-Skan Boundary-Layer Equation"

H. Chuang\*

University of Louisville, Louisville, Kentucky

THE author of Ref. 1 is to be commended for his efforts to improve the solution to the Falkner-Skan boundary-layer equation. The results shown in the paper are, however, a little bit misleading. It seems that the author did not thoroughly take into consideration the effects of  $\eta_\infty$  used in the calculation on the value of  $f''(0)$ , whose accuracy he was trying to improve. The method stated in the paper: "For  $-0.1988377 \leq \beta \leq -0.03$  the computed results are given to  $\eta = 10.00$ " will not yield the value of  $\beta = -0.1988377$  for  $f''(0) = 0$  and  $\eta_\infty = 10$ . Instead, the value of  $\beta = -0.1988448$  under the above-mentioned conditions. When  $\eta_\infty$  is increased to 12 and 14, both calculations will yield  $\beta = -0.1988378$  for  $f''(0) = 0$ . For  $\beta = -0.1987686895$  ( $m = -0.0904$ ), the values of  $f''(0)$  are found to be 0.0072985, 0.0049750, and 0.0047700 for  $\eta_\infty = 9.0, 10.0$ , and 12.0, respectively. When  $\eta_\infty$  is increased to 14.0, the value of  $f''(0)$  remains the same and is equal to 0.0047700. Therefore, the value of  $f''(0)$  has an error of 0.000205 for  $\eta_\infty = 10$ . However, the value of  $f'(\infty)$  will converge to 1 within the accuracy of  $10^{-7}$  for the case of  $\eta_\infty = 10.0$  and  $m = -0.0904$  provided that  $0.0049750 \leq f''(0) \leq 0.0049790$ . All of these calculations are based on the double-precision algorithms. It will never converge to the above-mentioned accuracy if  $f''(0)$  is set equal to 0.0047700 and  $\eta_\infty = 10.0$ .

It is found to be adequate to have  $\eta_\infty = 10.0$  for  $\beta = -0.06185567$  ( $m = -0.03$ ). But it is found to be unsatisfactory to have  $\eta_\infty = 7.5$  for  $\beta = -0.02$ . In this instance, the values of  $f''(0) = 0.3112713, 0.3112578$ , and  $0.3112577$  for  $\eta_\infty = 7.5, 9.0$ , and  $10.0$ , respectively. The value of  $f''(0)$  remains the same for  $\eta_\infty = 10$  and 11.

The boundary-layer thickness where  $u/U_\infty = 0.999$  will remain accurate to the second digit when the outer edge, where  $u/U_\infty = 1.0$ , is sufficiently large. Figure 1 shows the effects of the flow shape factor  $m = \beta/(2 - \beta)$  on the above-defined

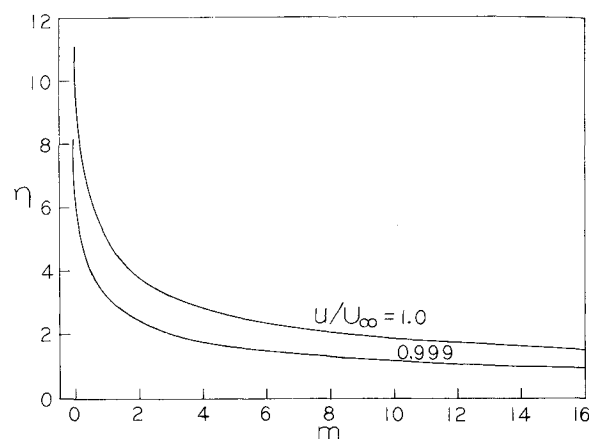


Fig. 1 Boundary-layer thickness vs flow shape factor.

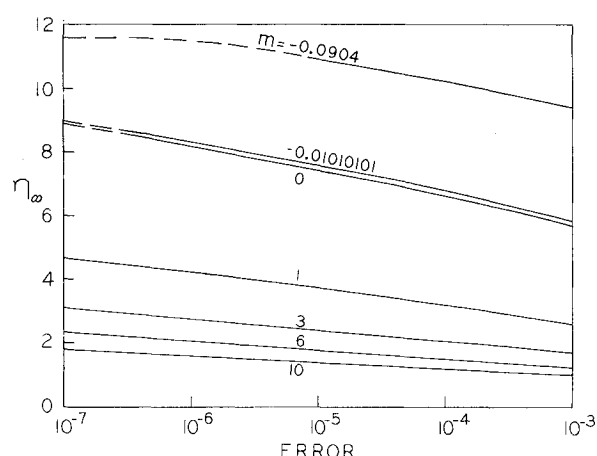


Fig. 2 Error in the dimensionless shear due to the outer edge location.

thickness and the least outer edge which must be used in the profile integration. There are, however, some exceptions. When  $m = 6$ , the least outer edge is 2.7, which yields  $f''(0) = 2.9383650$ . The outer edge from Fig. 1 is 2.4, which yields  $f''(0) = 2.9383651$ . The difference here is due to the roundoff effect. The thickness where  $u/U_\infty = 0.999$  remains the same at 1.5 for both calculations. Figure 2 shows the effects of the outer edge location on the accuracy of  $f''(0)$  for the cases where the value of  $f'(\infty)$  converges to 1 within the accuracy of  $10^{-7}$ . The error defined as the difference between the value of  $f''(0)$  at the given  $\eta_\infty$  and the convergent value as the outer edge approaches infinity. As mentioned before, the last two digits are not very accurate for  $m = -0.0904$  and the values of  $0.0047700 \leq f''(0) \leq 0.0047730$  will yield a solution with  $\eta_\infty \geq 11.75$ . If  $f''(0) = 0.0047700$  is used, a satisfactory solution can be obtained with the outer edge as low as 11.1. However, it is accurate to seven digits for all others where  $m > 0$ . The percent error increases with decreasing  $m$  as the value of  $f''(0)$  also decreases. Blottner<sup>2</sup> used the fourth-order finite difference scheme with various  $\eta_\infty$  and obtained a very accurate result for  $m = 0$  with only 81 grid points for  $\eta_\infty = 7$  and a convergence accuracy of  $10^{-10}$ . In comparison with the above results, the fourth-order Runge-Kutta method needs to integrate at least up to  $\eta_\infty = 9.5$  for  $m = 0$  and a convergence accuracy of  $10^{-7}$ . The former method seems to yield more accurate results for the skin friction with fewer grid points than the latter.

Equation (5) is inaccurate in the neighborhood of the point of flow separation,  $f''(0) = 0$ . At the flow separation point, the value of  $\beta$  should be equal to  $-0.1988378$  ( $m = -0.0904286$ ). Again, accuracies of Eqs. (5) and (6) are very poor. For example, Eq. (5) yields  $\beta = -0.103823$  while

Eq. (6) yields  $\beta = -0.107134$  for  $f''(0) = 0.3112577$ . Actually the value of  $f''(0)$  considered here corresponds with  $\beta = -0.02$  mentioned in the previous paragraph.

### References

<sup>1</sup>Forbich, C. A. Jr., "Improved Solutions to the Falkner-Skan Boundary-Layer Equation," *AIAA Journal*, Vol. 20, Sept. 1982, pp. 1306-1307.

<sup>2</sup>Blottner, F. G., "Introduction to Computational Techniques for Boundary Layers," Sandia National Laboratories, Albuquerque, NM, SAND79-0893, 1979.

### ERRATA

• "Simplified Implicit Block-Bidiagonal Finite Difference Method for Solving the Navier-Stokes Equations," Vol. 23, No. 7, 1985, pp. 1130-1132. The second author's name is V.S.V. Iyer.

• "Swirling Flow in a Research Combustor," Vol. 23, No. 2, 1985, pp. 241-248. The second author's name is H. T. Sommer, not Somer, as previously stated.

U.S. POST SERVICE STATEMENT OF OWNERSHIP, MANAGEMENT AND CIRCULATION (Required by 39 U.S.C. 3685)			
1. TITLE OF PUBLICATION <b>AIAA JOURNAL</b>		2. PUBLICATION NO. 1009920	3. DATE OF FILING OCT. 9, 1985
4. FREQUENCY OF ISSUE MONTHLY		5. NO. OF ISSUES PUBLISHED ANNUALLY 12	6. ANNUAL SUBSCRIPTION PRICE \$21.00
7. COMPLETE MAILING ADDRESS OF KNOWN OFFICE OF PUBLICATION (Street, City, County, State and ZIP Code; first priority)			
1633 BROADWAY, NEW YORK, N.Y. 10019			
8. COMPLETE MAILING ADDRESS OF THE HEADQUARTERS OF GENERAL BUSINESS OFFICES OF THE PUBLISHER (not printer)			
SAME AS ABOVE			
9. FULL NAMES AND COMPLETE MAILING ADDRESSES OF PUBLISHER, EDITOR, AND MANAGING EDITOR (Full name, street, city, state, and ZIP Code)			
PUBLISHER (Name and Complete Mailing Address) <b>AMERICAN INSTITUTE OF AERONAUTICS AND ASTRONAUTICS, INC.</b> SAME AS ABOVE			
EDITOR (Name and Complete Mailing Address) <b>GEORGE W. SUTTON</b> SAME AS ABOVE			
MANAGING EDITOR (Name and Complete Mailing Address) <b>ELIANT J. CAMBI</b> SAME AS ABOVE			
10. OWNER (If owned by a corporation, its name and address must be stated and also immediately thereunder the names and addresses of stockholders owning or holding 1 percent or more of total amount of stock. If not owned by a corporation, the names and addresses of the individual owners must be given. If owned by a partnership or other unincorporated firm, its name and address, as well as that of each individual must be given. If the publication is published by a partnership or other unincorporated firm, its name and address must be stated (If not, it must be completed).)			
FULL NAME <b>AMERICAN INSTITUTE OF AERONAUTICS AND ASTRONAUTICS, INC.</b>			
COMPLETE MAILING ADDRESS SAME AS ABOVE			
11. KNOWN BONDHOLDERS, MORTGAGEES, AND OTHER SECURITY HOLDERS OWNING OR HOLDING 1 PERCENT OR MORE OF TOTAL AMOUNT OF BONDS, MORTGAGES, OR OTHER SECURITIES (If none, so state)			
FULL NAME NONE			
COMPLETE MAILING ADDRESS NONE			
12. FOR COMPLETION BY NONPROFIT ORGANIZATIONS AUTHORIZED TO MAIL AT SPECIAL RATE (Section 3752, Internal Revenue Code) The purpose, function, and nonprofit status of the organization and the exempt status for Federal income tax purposes (Check one):			
<input checked="" type="checkbox"/> HAS NOT CHANGED DURING PRECEDING 12 MONTHS <input type="checkbox"/> HAS CHANGED DURING PRECEDING 12 MONTHS (If changed, publisher must submit explanation of change with this statement)			
13. EXTENT AND NATURE OF CIRCULATION		14. AVERAGE NO. COPIES EACH ISSUE DURING PRECEDING 12 MONTHS	
A. TOTAL NO. COPIES (Net Press Run)		5550	
B. PAID AND/OR REQUESTED CIRCULATION (Sum of paid and requested circulation, net of returns and exchange rates)		5159	
C. TOTAL PAID AND/OR REQUESTED CIRCULATION (Sum of B and C)		5159	
D. FREE DISTRIBUTION BY MAIL, CARRIER OR OTHER MEANS (Samples, complimentary, and other free copies)		94	
E. TOTAL DISTRIBUTION (Sum of D and E)		5065	
F. COPIES NOT DISTRIBUTED (1. Office use, left over, unsold, unclaimed, lost, etc. 2. Return from News Agents)		504	
G. TOTAL (Sum of E, F, and G) (Net Press Run (Sum of A and G))		5550	
I certify that the statements made by me above are correct and complete		SIGNATURE AND TITLE OF EDITOR, PUBLISHER, BUSINESS MANAGER, OR OWNER <b>CHRIS TROLL, CONTROLLER</b>	